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IMPROVING COMMERCIAL ACCOUNTING OF HYDROCARBONS TRANSPORTED BY PIPELINE

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It is shown that duplication of measurements by supplier and consumer is typical in commercial accounting of transported hydrocarbons. There is thus a problem of an imbalance in the measurement results that arises as a consequence of errors in the measuring instruments employed. Different approaches to the solution of this problem are analyzed mathematically and a technique and formulas for calculating the distribution of the imbalance are proposed.

Keywords: *hydrocarbons, commercial accounting, duplication of measurements, imbalance in measurement results, results of measurements by supplier and by customer, admissible risk.*

Duplication of measurements of the commercial parameters of a lot by the supplier and customer is typical in commercial accounting of hydrocarbons that are transported by pipeline [1]. This is explained by two circumstances, first the organization of stations for collection of data on hydrocarbons shipped by different enterprises or teams in the field and jointing of numerous local feed lines from consigners and branch pipes to the consignees in the trunk pipelines. In these cases, duplication of measurements is inevitable, since otherwise it is impossible to determine the fraction which each group contributes to the total volume of sold lots of product. Moreover, financial losses due to measurement errors in clearing accounting transactions always lead to duplication of measurements even if there is only a single shipper and a single customer in the technological transportation line.

Installation of their own accounting devices is advantageous to the parties in a transaction, since it makes it possible to monitor the other party's measurement results. However, in this case there arises the problem of imbalance in the results of measurements between those performed by the suppliers and those performed by the customers due to errors in the measurements by the measuring instruments used by the two parties, which may lead to substantial financial losses. For example, with the limit to the allowable error of measurements of the net mass of petroleum hydrocarbons being equal to 0.35%, the imbalance in the readings of two accounting points may reach 0.7%. When a tanker with 200,000 tons capacity (expressed in physical terms) is loaded, this amounts to 1400 tons of petroleum with value on the order of US$1,000,000, assuming a price for petroleum of US$90 per barrel.

In the planned management system, this did not create any sort of problems, since where there was a single owner (the state), it was sufficient to establish standards of imbalance (equal, for example, to the mean-square sum of the errors of the measurements carried out by the supplier and the customer) and where these standards were maintained, the imbalance could be described as production losses. In the case of market relations, transfer of product to a different legal entity is accompanied by a change in the ownership of the product. Unlike the readings of accounting devices, an imbalance in payments cannot occur. Therefore, establishing standards for factual losses of product does not solve the problem, since there arise a number of questions, for example, which party in a transaction should be made to assume ownership of these losses, the supplier or the customer? and how should these losses be divided between the parties in a transaction? In this connection, an imbalance in the readings of measuring instruments become the source of disagreements between parties in the market of

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energy carriers, which sometimes leads to insoluble situations of conflict and ultimately substantially reduces the quality of the pipeline transport services.

In actual practice, such contradictions are often eliminated in the following way. The readings of the measuring instrument at a higher precision class or possessing a verification certificate issued by a higher authoritative organization are recognized as the true readings. This undoubtedly is a powerful stimulus to increase the precision of commercial accounting, though methodologically such an approach is not justified or legitimate, inasmuch as both measuring instruments have contributed to the imbalance. Let us consider this question in more detail.

Measurement errors are generally random variables and are standardized by the limits of the admissible values which are symmetric relative to zero [2]. At the same time, the interests of the supplier and customer do not objectively coincide in this regard. It is more advantageous to the supplier if the measurement errors are positive and maximal within the range of admissible values. The customer, on the other hand, is interested in assurance that the measurements possess negative errors within the admissible limits and also that they are as great as possible in absolute value.

Let us denote by x_1 the relative error of the measurements performed by the supplier of a parameter P_i that influences the net mass of product (volume or mass flow rate, density, moisture content, etc.); x_2, relative error of measurements of this parameter performed by the customer; and δ_1 and δ_2, limits of admissible values of these errors. If the inequalities $-(\delta_1 + \delta_2) \le x \le \delta_1 + \delta_2$ are satisfied, this will not contradict the assumption that the errors of both of the measuring instruments are within the previously established limits [3]. Therefore, these inequalities correspond to the normal situation. To make the discussion concrete, suppose that the net mass grows with increasing P_i. If $x \le -(\delta_1 + \delta_2)$, the supplier will bear unwarranted losses, while if $x \ge \delta_1 + \delta_2$, the customer will bear these losses; if the net mass decreases where the first inequality is satisfied, the customer will bear unwarranted losses, and where the second inequality is satisfied, the supplier will bear unwarranted losses.

The principle of fairness and equality of the parties is fundamental to the method of division of the instrumental losses of hydrocarbons between parties in a market. Accordingly, each subject must provide compensation for the fraction of the total losses caused by the error of its accounting device alone. Thus, the contributions to the total imbalance attributable to each measuring instrument established in a supply – receiving point network must be estimated [4]. Since the values of the errors of these devices when in use are not known and only particular characteristics of their distribution by type are established, e.g., the limits of the admissible values or the mean values and the standard deviations, only the likely value of the error of each measuring instrument, not the exact contribution of this error, may be estimated. Thus, it is best to distribute the instrumental losses between the parties in a market in proportion to the relationship between the most likely values of the errors of their measuring instruments.

The mathematical expectation, or mean value taken over an entire general population, is generally considered such a value. Mathematical expectations are either unconditional and estimated a priori, or conditional and measured a posteriori [5]. If corrections to the readings of devices are determined prior to performing measurements, i.e., the problem then consists in a division of future instrumental losses, then because of the absence of any sort of additional information it becomes necessary to limit ourselves to a determination of an unconditioned mathematical expectation [6]. In this type of situation, such additional information exists and comprises the results of measurements performed by the supplier X_{1j} and the results of measurements performed by the customer X_{2j} and the imbalance in the readings:

$$x = \sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j},$$

where n_1 (respectively, n_2) is the number of accounting devices at the supplier (customer). Therefore, the conditional mathematical expectations of the errors of the accounting devices must be adopted as the most likely values of the errors of these devices, where these expectations are determined under the condition that the imbalance in the supply – receiving point system is equal to some known value x.

The conditional mathematical expectations for the jth supplier device and for the jth customer device are denoted correspondingly as

$$y_{1j} = M\left(x_{1j} / \left(\sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j}\right) = x\right);$$

$$y_{2j} = M\left(x_{2j} / \left(\sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j}\right) = x\right).$$

We consider that $X_{1j} = X_{1jf} + x_{1j}$; $X_{2j} = X_{2jf} + x_{2j}$, where X_{1jf} and X_{2jf} are the factual results of the measurements while x_{1j} and x_{2j} are the errors of these measurements. Since

$$\sum_{j=1}^{n_1} X_{1jf} = \sum_{j=1}^{n_2} X_{2jf},$$

it is obvious that

$$\sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j} = \sum_{j=1}^{n_1} x_{1j} - \sum_{j=1}^{n_2} x_{2j},$$

hence

$$y_{1j} = M\left(x_{1j} / \left(\sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j}\right) = x\right) = M\left(x_{1j} / \left(\sum_{j=1}^{n_1} x_{1j} - \sum_{j=1}^{n_2} x_{2j}\right) = x\right);$$

$$y_{2j} = M\left(x_{2j} / \left(\sum_{j=1}^{n_1} X_{1j} - \sum_{j=1}^{n_2} X_{2j}\right) = x\right) = M\left(x_{2j} / \left(\sum_{j=1}^{n_1} x_{1j} - \sum_{j=1}^{n_2} x_{2j}\right) = x\right).$$

Let us consider the above dependences. To simplify the discussion, we will first make the inessential assumption that there exists only a single device at the supplier and only a single device at the customer. Next, it is natural to assume that the errors of these devices are random variables distributed over the population of measuring instruments of the indicated types in accordance with a normal law.

Let $f_1(x_1)$ and $f_2(x_2)$ denote the distribution densities of the errors of the supplier's and of the customer's accounting device and m_1, m_2, σ_1, and σ_2 the mean values and standard deviations of these distributions, respectively. The conditional mathematical expectations $M(x_1 | x_2 - x_1 = x)$, $M(x_2 | x_2 - x_1 = x)$ of the errors of the accounting devices x_1 and x_2 under the condition $x_2 - x_1 = x$ must be found.

The condition $x_2 - x_1 = x$ is equivalent to the condition $x_2 = x + x_1$.Therefore, by definition, the conditional mathematical expectation is expressed as

$$y_1 = M(x_1 | x_2 - x_1 = x) = M(x_1 | x_2 = x + x_1) = \frac{\int_{-\infty}^{\infty} x_1 f_1(x_1) f_2(x + x_1) dx_1}{\int_{-\infty}^{\infty} f_1(x_1) f_2(x + x_1) dx_1}.$$

Substituting into the latter expression the formula for the density of a normal distribution

$$f_i(x) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[(x - m_i)^2 / \sigma_i^2\right], \quad i = 1, 2,$$

we obtain

$$y_1 = \frac{m_1\sigma_2^2 + (m_2 - x)\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = \frac{m_1 - [x - (m_2 - m_1)]\sigma_1^2}{\sigma_1^2 + \sigma_2^2}. \tag{1}$$

Similarly, we find

$$y_2 = M(x_2 \,|\, x_2 - x_1 = x) = M(x_2 \,|\, x_1 = x_2 - x) = \frac{\int_{-\infty}^{\infty} x_2 f_2(x_2) f_1(x_2 - x) dx_2}{\int_{-\infty}^{\infty} f_2(x_2) f_1(x_2 - x) dx_2} =$$

$$= \frac{m_2\sigma_1^2 + (m_1 + x)\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{m_2 + [x - (m_2 - m_1)]\sigma_2^2}{\sigma_1^2 + \sigma_2^2}. \tag{2}$$

Let us check these results. The difference between the most likely values of the measurement errors must be equal to the imbalance x:

$$y_1 - y_2 = \frac{m_2 - m_1 + (x - m_2 + m_1)(\sigma_1^2 + \sigma_2^2)}{\sigma_1^2 + \sigma_2^2} = x. \tag{3}$$

These formulas are easily generalized to the case of the set of measuring instruments at the suppliers and at the customers:

$$y_{1j} = m_{1j} - \left[x - \sum_{j=1}^{n_2} m_{2j} + \sum_{j=1}^{n_1} m_{1j} \right] \sigma_{1j}^2 S_\Sigma^{-2}; \tag{4}$$

$$y_{2j} = m_{2j} + \left[x - \sum_{j=1}^{n_2} m_{2j} + \sum_{j=1}^{n_1} m_{1j} \right] \sigma_{2j}^2 S_\Sigma^{-2}, \tag{5}$$

where

$$S_\Sigma^2 = \sum_{i=1}^{n_2} \sigma_{2i}^2 + \sum_{i=1}^{n_1} \sigma_{1i}^2$$

is the variance of the likely imbalance of the results of all the measurements.

At the same time, the probability of metrological failure of a measuring instrument cannot be excluded, especially when the absolute value of the deviations is greater than the sum of the limits of the admissible values of the errors of their devices.

Once it is established that one of the measuring instruments is in fact metrologically malfunctioning, its readings are acknowledged as being legally trivial and a calculation of the transferred product is performed on the basis of the readings of the second device. In actual practice, this may be demonstrated only by a special verification of the measuring instruments performed upon the decision of an arbitration court. It is clear that recourse to an arbitration court is necessary only when the probability of a failure of the opposing party's measuring instrument is high, since if the actuality of its metrological malfunctioning is not confirmed, the side that has resorted to a court with a lawsuit bears additional losses for payments for legal costs, including the cost of verification. Thus, computational dependences for use in determining the conditional probabilities P_1 and P_2 of metrological faults in the supplier's or in the customer's accounting device under the condition that the imbalance is equal to x are of practical interest. Under the above assumption of a normal distribution of the errors of a measuring instrument,

$$P_1 = \Phi\left[\frac{(x + m_2 - \delta_1)\sigma_1^2 - (\delta_1 - m_1)\sigma_2^2}{\sigma_1\sigma_2\sqrt{\sigma_1^2 + \sigma_2^2}}\right] + \Phi\left[-\frac{(x + m_2 + \delta_1)\sigma_1^2 + (\delta_1 + m_1)\sigma_2^2}{\sigma_1\sigma_2\sqrt{\sigma_1^2 + \sigma_2^2}}\right];$$

$$P_2 = \Phi\left[\frac{(x - m_1 - \delta_2)\sigma_2^2 - (\delta_2 + m_2)\sigma_1^2}{\sigma_1\sigma_2\sqrt{\sigma_1^2 + \sigma_2^2}}\right] + \Phi\left[-\frac{(x - m_1 + \delta_2)\sigma_2^2 + (\delta_2 - m_2)\sigma_1^2}{\sigma_1\sigma_2\sqrt{\sigma_1^2 + \sigma_2^2}}\right],$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-0.5y^2}\,dy$$

is the integral Laplace function.

If $m_1 = m_2 = 0$, $\sigma_1 = \delta_1/3$, and $\sigma_2 = \delta_2/3$, we have

$$P_1(y, k) = \Phi\left[3\frac{y - \sqrt{1 + k^2}}{k}\right] + \Phi\left[-3\frac{y + \sqrt{1 + k^2}}{k}\right];$$

$$P_2(y, k) = \Phi\left[3\left(yk - \sqrt{1 + k^2}\right)\right] + \Phi\left[-3\left(yk + \sqrt{1 + k^2}\right)\right];$$

$$y = x\Big/\sqrt{\delta_1^2 + \delta_2^2};$$

and $k = \delta_2/\delta_1$. Hence, it follows that it is sufficient to construct a graph of one of these dependences, since $P_2(y, k) = P_1(y, 1/k)$.

Once graphs of $P_2(y, k)$ have been constructed for $k = 0.2$–5, it may be shown, for example, that the probability of a metrological fault in the customer's measuring instrument is at least 0.99 with $k = 2$ and that $|x| \geq 1.5(\delta_1^2 + \delta_2^2)^{1/2}$ [7]. Therefore, simple practical recommendations for arriving at decisions on the basis of results of comparisons of the readings of the measuring instruments may be developed. Thus, if the imbalance is 150% of the mean-square sum of the limits of the admissible errors of the measuring instrument, the party to the transaction possessing twice as precise a measuring instrument may turn to an arbitration court with confidence, since its risk will then amount to only 0.01.

The sequence of steps will then be as follows:

1) specify the value of the admissible risk α;

2) find from the graph the minimal values of the imbalance for acknowledgement that a supplier's measuring instrument is faulty $x_{min}(k, P_2 = 1 - \alpha)$ or that a customer's measuring instrument is faulty $x_{min}(1/k, P_1 = 1 - \alpha)$;

3) the next steps are determined by the relationship between x and x_{min}. Thus, if $x \geq x_{min}$ ($x \leq -x_{min}$), the supplier (respectively, customer) resorts to an arbitration court with a lawsuit demanding acknowledgement that the measurement results of the other party to the transaction are wrong, and if this inequality is not satisfied, the other party agrees to a division of the imbalance in accordance with formulas (4) and (5).

From formulas (1)–(5), the corrections to the readings of the measuring instruments may be calculated when there exist statistical data on the results of verification of the instrument that enable us to determine the mean value m_j and the standard deviation σ_j of the error of the measurements performed by each accounting device. However, these data are most often lacking. In such cases, information about the precision standards of the measuring instruments that are being employed, that is, data specified in the certificates or in specifications on the limits of the admissible errors δ_j, may be used to calculate the corrections.

The first method of dividing the imbalance between the parties in a transaction, where the distribution of the imbalance is directly proportional to the limits of the admissible errors of the accounting devices, was developed on the basis of this information. In this case, the corrections to the supplier's and the customer's readings are found thus:

$$y_1 = -x\delta_1/(\delta_1 + \delta_2); \quad y_2 = x\delta_2/(\delta_1 + \delta_2), \tag{6}$$

and then the corrected measurement results:

$$X_1^* = X_1 - x\delta_1/(\delta_1 + \delta_2); \quad X_2^* = X_2 + x\delta_2/(\delta_1 + \delta_2), \tag{7}$$

Such an approach is used especially when computing the magnitude of the mass of petroleum delivered by oil-producing enterprises to oil trunk pipelines [8], though the errors of the measurements performed by all the parties involved in transfer of the product are taken into account here. From the scientific point of view, however, this method does not hold up to criticism. Let us show how it is unsound.

From formulas (6) it follows that $y_1/y_2 = \delta_1/\delta_2$. Hence, the measurement errors $y_1 = k\delta_1$; $y_2 = k\delta_2$, where k is a constant coefficient that is the same for both measuring instruments, and δ_1 and δ_2 are constant values. Thus, both measuring instruments have only systematic errors the ratio between which is precisely equal to the ratio of the limits of their admissible errors.

In the general case, the errors of the measuring instruments are random variables, since they contain a systematic and a random component and, moreover, as to the systematic components it is known only that these components have a distribution over all measuring instruments of the same type in accordance with some probability law, i.e., are also random variables. Therefore, the hypothesis that k is a constant coefficient may be discarded at once.

The errors of the measuring instruments are known to consist of calibration errors and errors due to ageing of the elements of the measuring instruments [9]. The values of both components of the error are determined by a set of random factors and are therefore unique to each specimen of a measuring instrument. Consequently, the hypothesis that k is a random variable and at the same time is of the same magnitude for different measuring instruments cannot be justified on physical grounds.

There exists a different approach that is more justified. Since the precision standards of the measuring instruments are specified in the form of limits of the admissible error that are symmetric relative to zero, it may be assumed that the mean value of the error is equal to zero. Therefore, we may set $m_j = 0$ for all $j = 1, 2,\ldots, n$ ($n = n_1 + n_2$). The limits of the admissible values of the controlled parameters are generally established in engineering as a function of the standard deviation of the distribution of this parameter in accordance with the relationship $\delta_j = 3\sigma_j$ [10]. With this in mind, we substitute $\sigma_j = \delta_j/3$ into (4) and (5), and these formulas then assume the form

$$y_{1j} = -x\delta_j^2 \big/ \delta_\Sigma^2, \quad y_{2j} = x\delta_j^2 \big/ \delta_\Sigma^2, \quad j = 1, 2, ..., n_2, \tag{7}$$

where

$$\delta_\Sigma = \sqrt{\sum_{j=1}^{N} \delta_j^2}$$

is the mean-square sum of the limits of the admissible errors of all the accounting devices installed in the network.

In this case, the imbalance is divided between the supplier and the customer in proportion to the squares of the precision classes of their devices.

Estimates that are arrived at on the basis of formulas (6) and (7) will differ significantly. For example, suppose that the measurement precision of the supplier's measuring instruments is three times that of the customer's instruments: $\delta_1 = \delta_2/3$. A calculation of the supplier's and customer's fraction performed according to formula (6) will give $y_1 \approx 0.25x$, $y_2 \approx 0.75x$, and with the use of formula (7), $y_1 = 0.1x$, $y_2 = 0.9x$. Consequently, in this example the error associated with the determination of these fractions due exclusively to the single factor of uncertainty of the relationship between the mean value and standard deviation of the distribution of the error over a set of measuring instruments of the same type may reach 15% of the total imbalance. At the same time, it is necessary to bear in mind that the actual relationship between the limit of the admissible error and these characteristics of the errors of particular measuring instruments is not known. Thus, the factual error in the determination of the fractions of the imbalance performed by means of (6) or (7) may be substantially greater and in limit attain 100%.

A more precise approach entails recourse to additional measurement information for redetermination of the mean values m_i and the standard deviations σ_i of the errors of the measuring instruments. Since measurements of flow rate in transfer and acceptance of petroleum, petroleum products, and natural gas are performed in continuous fashion, there is a considerable quantity of such data available. For example, if the transfer-acceptance petroleum balance is calculated once per month, then, instead of a single monthly imbalance equation a system of daily imbalance equations may be compiled:

$$x_j = Q_{1j} - Q_{2j}, \qquad j = 1, \ldots, N, \tag{8}$$

where Q_{1j} and Q_{2j} are the results of measurements of the volume of petroleum delivered on the jth day performed by the supplier and by the customer, respectively; and N is the number of days in the reporting month, moreover, the monthly imbalance will be

$$x = \sum_{j=1}^{N} x_j .$$

Next, in addition to the daily imbalances Q_{1j} and Q_{2j} let us also estimate their absolute measurement errors ΔQ_{1j} and ΔQ_{2j}. If the measurements are performed by means of restrictions, the estimates of the errors will be found in accordance with a technique regulated by the standard [13]. Then the coefficients of the interpolating polynomials for the functions $\Delta Q_1 = a_1 + b_1 Q_1 + c_2 Q_1^2$ and $\Delta Q_2 = a_2 + b_2 Q_2 + c_2 Q_2^2$ and estimates of their variances are established by the method of least squares.

To simplify the discussion, we will consider the proposed technique for the case of a single supplier and a single customer in a transportation chain. Since the result of measurements of the quantity of petroleum hydrocarbons $Q_{ij} = Q_{ij}^* + m_i Q_{ij}^* \cong Q_{ij}^* + m_i Q_{ij}$ (Q_{ij}^* is the factual value of the quantity of petroleum measured by the ith measuring instrument on the jth day and m_i the relative error of the ith measuring instrument), and since the equality $Q_{1j}^* = Q_{2j}^*$ is satisfied where there are no losses of petroleum in the course of transportation of the petroleum from the supplier to the customer, the system of equations (8) assumes the form

$$m_1 Q_{1j} - m_2 Q_{2j} - x_j = \nu_j, \qquad j = 1, \ldots, N, \tag{9}$$

where ν_j is the error of closure of the jth equation due to errors in measurements of Q_{1j} and Q_{2j}.

The most probable values of the relative measurement errors m_i are found in accordance with Legendre's principle from the minimum condition imposed on the sum of the squares of the errors of closure of all the equations:

$$L = \sum_{j=1}^{N} \nu_j^2 = \sum_{j=1}^{N} \left[m_1 Q_{1j} - m_2 Q_{2j} - x_j \right]^2 = \min .$$

Hence, a system of equations $\partial L/\partial m_1 = 0$ and $\partial L/\partial m_2 = 0$ is obtained, and this system is transformed into a system of linear algebraic equations:

$$\left.\begin{aligned} S_{11}m_1 - S_{12}m_2 &= Y_1; \\ S_{12}m_1 - S_{22}m_2 &= Y_2, \end{aligned}\right\} \tag{10}$$

where

$$S_{11} = \sum_{j=1}^{N} Q_{1j}^2; \quad S_{12} = \sum_{j=1}^{N} Q_{1j}Q_{2j}; \quad S_{22} = \sum_{j=1}^{N} Q_{2j}^2; \quad Y_1 = \sum_{j=1}^{N} Q_{1j}x_j; \quad Y_2 = \sum_{j=1}^{N} Q_{2j}x_j.$$

A solution of system (10) yields estimates of the errors of the measuring instruments that are in best agreement with the results of the actual measurements:

$$\tilde{m}_1 = D_1/D; \qquad \tilde{m}_2 = D_2/D,$$

where

$$D = -\sum_{\substack{j,l=1,\ldots,N,\\ j\neq l}} (Q_{1j}Q_{2l} - Q_{1l}Q_{2j})^2;$$

$$D_1 = -\left[\sum_{j=1}^{N} Q_{2j}^2 \sum_{j=1}^{N} Q_{1j}x_j - \sum_{j=1}^{N} Q_{1j}Q_{2j} \sum_{j=1}^{N} Q_{2j}x_j\right];$$

$$D_2 = -\left[\sum_{j=1}^{N} Q_{1j}^2 \sum_{j=1}^{N} Q_{2j}x_j - \sum_{j=1}^{N} Q_{1j}Q_{2j} \sum_{j=1}^{N} Q_{1j}x_j\right].$$

The deviations of the factual values of the errors of the measuring instruments from their estimates $\tilde{m}_1$, $\tilde{m}_2$ are characterized by the standard deviations

$$\tilde{\sigma}_1 = S\sqrt{\sum_{j=1}^{N} Q_{2j}^2 \Big/ \sum_{j,l=1,\ldots,N,j\neq l} (Q_{1j}Q_{2l} - Q_{1l}Q_{2j})};$$

$$\tilde{\sigma}_2 = S\sqrt{\sum_{j=1}^{N} Q_{1j}^2 \Big/ \sum_{j,l=1,\ldots,N,j\neq l} (Q_{1j}Q_{2l} - Q_{1l}Q_{2j})},$$

where

$$S = \sqrt{\sum_{j=1}^{N} \nu_j^2 \Big/ (N-4)}$$

is an estimate of the standard deviation of the error of system of equations (10) and ν_j is determined from (9) following substitution of $\tilde{m}_1$, $\tilde{m}_2$.

The fractions of the imbalance due to errors in the supplier's and customer's measuring instruments are found following substitution of the values of $\tilde{m}_1$, $\tilde{m}_2$, $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ into (1), (2):

$$y_1 = \tilde{m}_1 Q_1 - [x - (\tilde{m}_2 Q_2 - \tilde{m}_1 Q_1)] \sum_{j=1}^{N} Q_{1j}^2 Q_2^2 \left(\sum_{j=1}^{N} Q_{1j}^2 Q_2^2 + \sum_{j=1}^{N} Q_{2j}^2 Q_1^2 \right)^{-1};$$

$$y_2 = \tilde{m}_2 Q_2 + [x - (\tilde{m}_2 Q_2 - \tilde{m}_1 Q_1)] \sum_{j=1}^{N} Q_{2j}^2 Q_1^2 \left(\sum_{j=1}^{N} Q_{1j}^2 Q_2^2 + \sum_{j=1}^{N} Q_{2j}^2 Q_1^2 \right)^{-1}, \tag{11}$$

where

$$Q_i = \sum_{j=1}^{N} Q_{ij}, \quad i = 1, 2.$$

From (11) it follows that in the proposed method of calculation, the results of measurements of the volumes of hydrocarbons shipped by the suppliers and received by the customers at different time intervals are the only initial data. Thus, in creating a distribution of the imbalance we take into account not the standardized limits of the admissible errors, which are the precision characteristics of the entire stock of measuring instruments of a given type, which, moreover, may be extremely far from the factual errors of actual specimens of these instruments, but instead factual errors, information about which is contained in the results of the performed measurements. This is a significant advantage of the present method, which creates prerequisites for the most precise and most legitimate distribution of the imbalance in the readings between the supplier and the customer.

In such an approach, information about the limits δ_i of the admissible errors may also be used to solve another problem, that having to do with monitoring the precision of the measuring instruments. In fact, if the condition $|\tilde{m}_i + 3\tilde{\sigma}_i| \le \delta_i$ is satisfied, then with probability close to 1, the factual measurement error will not exceed the already established standards. But if this condition is not satisfied, an excursion of the error of the measuring instruments beyond the admissible limits may be expected and, consequently, a special check of the instrument will have to be performed.

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